# AN APPROXIMATE METHOD FOR THE CALCULATION OF HEAT AND MASS EXCHANGE BETWEEN GAS AND A LIQUID FILM

E. I. Andreev

UDC 536.24

Using integral relations between the velocity and enthalpy of gas, we develop an engineering method for the calculation of the local and average parameters of media and the heat and mass flux without using the empirical data about contact instruments.

The mutually related heat and mass transfer between gas and liquid during their direct contact are usually treated theoretically by using a standard system of differential equations for the transfer of momentum, mass and energy in the boundary layer [1]. Since there is no information about how the temperature and concentration change along the surface of the contact one uses, alongside other boundary conditions, two tests equations (according to the amount of unknown gas parameters) which reflect the experimental data. As a rule, these test equations are of individual nature and have a narrow region of application.

Andreev [2] developed a method for treating theoretically the heat and mass exchange during steady concurrent laminar flows (single flow) of gas and a film of liquid with a variable temperature in a planar channel without the use of empirical data and the test conditions. The essence of the method lies in a transformation of the differential equations into integral ones. These equations are solved for the appropriate boundary conditions on one of the small segments $\Delta x_1$ of the surface of the channel into which the channel is decomposed. Within each of these segments, the parameters of the media in the longitudinal direction (along the flow of the gas in the direction of the x axis) can be assumed constant. Consequently, starting from the first segment $\Delta x_1$ at the entrance of the gas into the channel, if the initial parameters of the media are known (which are constant for $\Delta x_1$), the solution of the integral equation is used to determine the thickness $\delta_w$ of the thermal boundary layer (with respect to enthalpy) of the gas for $\Delta x_i$, and mass (vapor) and heat fluxes through the contact surface between the phases:

$$G_{vi} = D\frac{\rho_{vw}-\rho_{vf}}{\delta_w}\, b\Delta x_i,\quad Q_i = \left(\lambda\frac{t_w-t_f}{\delta_w} + D\frac{\rho_{vw}-\rho_{vf}}{\delta_w}\, I_v\right) b\Delta x_i.$$

The initial parameters of the media, their flow rates and the mass and heat fluxes are then used to determine, on the segment $\Delta x_i$, the final parameters of the media on this segment which are the initial values for the next segment (i + 1). The calculation is then repeated.

Because the liquid is considerably higher than the thermal conductivity of the gas, the temperature change in the liquid film along the transverse direction can be neglected in the first approximation. The temperature can therefore be assumed constant and equal to the average temperature $t_f$ within each segment of the channel. At the same time, the change of temperature of the liquid in the longitudinal direction is taken into account by the heat balance equation $t_{ef} = t_f \pm Q_i/(G_f c_f)$. The plus sign corresponds to a temperature increase of the liquid along the motion of the gas, and the minus sign corresponds to a decrease. The parameters and the mass and heat fluxes along the channel are represented in the form of piecewise constant functions. The accuracy with which this approximation reflects the realistic smoothly varying functions improves as the size of the chosen computation step (the length of the segment of the channel $\Delta x_i$) decreases. The total heat and mass fluxes are equal to $Q=\sum_i Q_i,\ G_v=\sum_i G_{vi}$ .

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Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 53, No. 2, pp. 191-198, August, 1987. Original article submitted April 28, 1986.

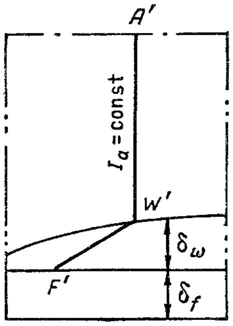

Fig. 1

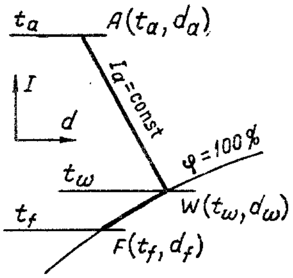

Fig. 2

Fig. 1. Enthalpy profile in the thermal boundary layer and in the core of the gas flow.

Fig. 2. A diagram of the enthalpy profile in the I–d diagram of the gas.

Separately, one should mention two new boundary conditions which are used instead of the test conditions. Independently of the method of solution for the system of differential equations, these boundary conditions play a fundamental role since they eliminate the necessity of using empirical data.

The essential principles of one of these conditions can be seen by considering the part of the channel which contains the liquid film and gas flow (Fig. 1), and a change of the state along the thickness of the channel in the I–d diagram of the humid gas (Fig. 2). First we investigate a linear distribution of the parameters in thermal (enthalpy) boundary layer in contact with the liquid. The line A'W' (Fig. 1) of constant enthalpy I = constant of the thermal core in the I–d diagram (Fig. 2) is represented by an identical straight line AW. The straight line of the enthalpy profile F'W' begins in the boundary layer of the saturated gas adjacent to the surface of the liquid. Consequently, in the I–d diagram, this corresponds to a segment of the line FW of the saturated gas close to the straight line (Fig. 1). The point of intersection W of the two straight lines (Fig. 2) is identical to the point W' (Fig. 1). Since the point W lies on the saturation line, the gas is saturated also in the point W'. Furthermore, away from point W towards point A, the gas is unsaturated, i.e., the point W and, accordingly, also point W' belong to the gas saturation boundary. The geometrical locus of points where the lines of equal enthalpy in the core (Fig. 1) intersect the lines of variable enthalpy in the boundary layer is the saturation boundary of the gas, i.e., the gas in the thermal (enthalpy) boundary layer is saturated, and outside the layer, it is unsaturated. The temperature $t_w$ of the saturation boundary as a collection of points W on each segment $\Delta x_i$ along the entire surface contact is equal to the gas temperature measured by a wet thermometer (Fig. 2).

It is known that the change of state of the gas between two points in the I–d diagram, when the parameters in these points are constant, is represented by a straight line [3], i.e., the rectilinearity of the line is exact and independent of the form of the profile F'W', linear or nonlinear. Thus, in the thermal (enthalpy) boundary layer of the gas, its boundary which is the natural saturation boundary assumes the temperature $t_w$ of the gas measured by a wet thermometer. The thermal (enthalpy) boundary layer is therefore also a layer of saturated gas.

As a result, the conditions at the saturation boundary can be written as

$$y=\delta_w,\ t=t_w,\ d=d_w,\ \rho_v=\rho_{vw},\ I=I_w=I_a. \tag{1}$$

The other boundary condition is the analytical equation for the intensity of the heat and mass exchange

$$\Delta_t=\Delta_d. \tag{2}$$

A considerable number of works has been devoted to derive and prove this equation [2]. We shall show that (2) expresses the fact that the average temperature and concentration pressures, divided by the maximum possible values, are equal on the heat-exchange segment under consideration. For an arbitrary scheme of relative motion of the gas and liquid (single flow, counterflow, mixed flow), the computational dependences are identical:

$$\Delta_t = \Delta t/\Delta t_{\max},\ \Delta_d = \Delta d/\Delta d_{\max},\ \Delta t_{\max} = t_w - t_{bf},\ \Delta d_{\max} = d_w - d_a,$$

$$\Delta t = \frac{t_w + t_{ew}}{2} - \frac{t_f + t_{ef}}{2},\ \Delta d = \frac{d_w + d_{ew}}{2} - \frac{d_a + d_{ea}}{2}.$$

A detailed experimental test confirms the validity and applicability of boundary conditions (1) and (2) for any contact instrument and for the heat and mass exchange processes in it.

The boundary condition (1) and the fact that the saturation boundary assumes the temperature of the gas measured by the wet thermometer give an improved distribution of the gas and liquid parameters in the boundary layers which is shown in Fig. 3 for processes of different direction. Since the relationship between the parameters of saturated gas is one-to-one, the graphs of their change have slopes of the same sign. For example, in an adiabatic process, the graphs of t, d, and I in the boundary layer of saturated gas are perpendicular to the boundary of the layer (Fig. 3a). Beyond the limits of the saturated gas, i.e., in the boundary layer of unsaturated gas, the enthalpy is constant according to (1). If one follows, in the I–d diagram, the lines of constant enthalpy WA (Fig. 2), it is clear that the moisture content of the gas decreases, and the temprature of the gas simultaneously increases (see Fig. 3a, graphs of t and d, in the layer of unsaturated gas).

During the cooling of the gas (Fig. 3b), the lines t, d, and I have a slope of the same sign in the layer of saturated gas and, for a constant enthalpy, the lines of t and d have different directions, analogously to the adiabatic process. It is seen that inside the boundary layer (at the boundary between the layers of saturated and unsaturated gas), there is an extremum of d (see also Fig. 2). In the case of heating of the gas (Fig. 3c), an extremum of temperature appears inside the layer. These extrema are known and are, as it is seen, a result of the mutual effect between the heat and mass exchange [4-6].

In contrast with the temperature in the boundary of layer of saturated gas which is assumed, in some cases, constant and equal to the temperature of the liquid [3], the present improvement of the accuracy of the distribution of t, d, and I in the boundary layer of the gas makes it possible to take into account the change of these quantities in the calculation of polytropic processes of heat and mass exchange. The counterflow of gas and liquid for known initial parameters of the media is calculated analogously [1], i.e., by iterations, and it is the final temperature of the liquid which is given. If the initial parameters of the gas and the final temperature of the liquid (at the entrance of the gas into the instrument) are known, the calculation is carried out directly from these values.

For a turbulent gas flow, the method of calculation has a number of specific features. One uses the known empirical data about turbulent boundary layers. In particular, the distribution of parameters is taken in the form of a power law with exponent $n = 1/7$. The thickness of the viscous sublayer was determined using the constant $S = \delta_m v_*/\nu = 5$ [7]. The constant of turbulence $\kappa_0 = 0.048$ for a planar channel with wetted walls was chosen empirically with allowance for the analogous constant in the wake flow behind a moving body which is equal to 0.047 [7].

At the entrance to the channel, there is a segment with a laminar boundary layer of gas where all the calculations are carried out by the method of [2]. Together with the linear enthalpy distribution in the boundary layer $I - I_f = (I_w - I_f)y/\delta_w$ the calculation can be simplified, using the fact that the thickness of the laminar layer is considerably smaller than the thickness of the channel, by assuming that the gas velocity in the layer changes linearly according to $u = u_a y/\delta$. From the balance equations for the enthalpy and for the mass in the final cross section of the computational segment $\Delta x_i$

$$I_2 G_g = b\rho_g \int_0^{\delta_w} Iudy + G_a I_{ea},\ G_a = G_g - G_b,\ G_b = b\rho_g \int_0^{\delta_w} udy$$

the enthalpy of the gas in the core of the flow can be found from the expression

$$I_{ea} = \frac{I_2 G_g - (1/3)\,G_b I_{ef}}{G_g - (1/3)\,G_b}.$$

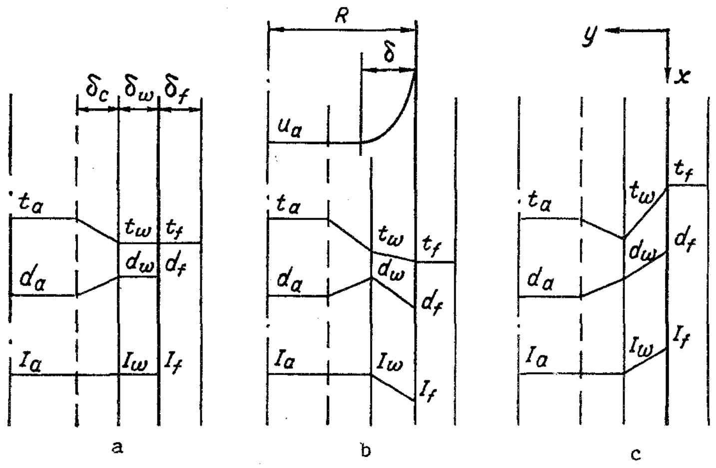

Fig. 3. Profiles of the enthalpy of the gas and of the potentials of the heat and mass transfer in the boundary layers of the gas and liquid. a) The adiabatic process; b) cooling of the gas; c) heating of the gas.

The boundary layer can be assumed laminar up to $\mathrm{Re}_{cr} = u_a\delta_1/\nu \leq 575$ [7]. Beyond this condition, the hydrodynamic boundary layer is assumed to be turbulent. The transition segment is omitted in view of its relatively small extent, and because of the absence of reliable methods of calculation. The thickness of the layer will be calculated using the formula [7]:

$$\delta = \left[\delta_0^{5/4} + 0{,}289\left(\frac{\nu}{u_a}\right)^{1/4}\Delta x_i\right]^{0,8}.$$

The thickness of the thermal layer of the gas $\delta_w$ will be determined by the method [2] without any changes.

The characteristic feature of the calculation below is that the heat and mass fluxes on the segment $\Delta x_i$ are found by using the difference of parameters on the inner and outer boundaries of the viscous sublayer:

$$Q_i = \left(\lambda_\Sigma \frac{t_{wm} - t_f}{\delta_m} + D_\Sigma \frac{\rho_{vwm} - \rho_{vf}}{\delta_m} I_v\right) b\Delta x_i.$$

The distributions of the parameters in the segment are assumed linear. The temperature on the outer boundary of the sublayer is found with allowance for its power-law distribution in the layer as a whole:

$$t_{wm} = t_f + (t_w - t_f)(\delta_m/\delta_w)^{1/7}.$$

The thickness of the viscous sublayer is determined from the formula

$$\delta_m = \frac{S\nu}{\bar{u}} \sqrt{\frac{\bar{u}d_e}{8\nu}}$$

using the above constant $S = \delta_m\nu_*/\nu = 5$ and the following formulae: for the dynamical velocity of gas near the wall or boundary with the liquid $\nu_* = u\sqrt{\lambda_{fr}/8}$, for the friction coefficient in a laminar flow $\lambda_{fr} = 64/\mathrm{Re}$, and the Reynolds number $\mathrm{Re} = ud_e/\nu$. The total molecular and turbulent heat conductivities and the diffusion constant will be averaged over the thickness of the sublayer, and will be calculated according to the formulae $\lambda_\Sigma = \lambda + \lambda_T$, $D_\Sigma = \Psi D + D_T$. The velocity gradient within the limits of the viscous sublayer will be found by using the method [2] by assuming that the coefficient of dynamical viscosity is constant. Then

$$u_m = \frac{3}{2} u_a \frac{\delta_m}{\delta}.$$

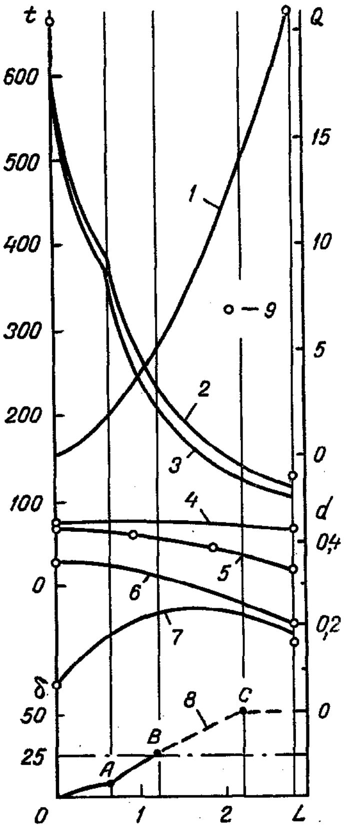

Fig. 4. Theoretical parameters of the gas and liquid: 1) total heat flux transferred from the gas to liquid; 2) temperature of the gas measured by a dry thermometer at the axis of the channel; 3) average temperature of the gas measured by a dry thermometer; 4) average temperature of the gas measured by a wet thermometer; 5) average temperature of the liquid; 6) absolute humidity of the gas (at temperature 4); 7) absolute humidity of the gas (at temperature 2); 8) thickness of the hydrodynamic boundary layer of the gas (the points A, B, and C are, respectively, the thoretical point where the laminar layer transforms into a turbulent one, the point where the opposite layers meet on the axis of the channel, and where a layer reaches the opposite wall); 9) experimental points. The quantity t is in deg C, δ in mm, Q in kW, d in kg/kg, and L in m.

The turbulent diffusion constant will be determined from the Prandtl hypothesis $D_T = \kappa_1 \delta_m u_m$. The coefficient $\kappa_1$ for a developed turbulent flow is a constant quantity, but it is variable for a nondeveloped flow. Indeed, in the transition point between a laminar layer and a turbulent one (for $Re_{cr} \approx 575$), the calculated turbulent fluctuations are absent, i.e., $\kappa_1 = 0$. As the turbulent layer develops, the coefficient changes in proportion to the increase of the thickness of the layer:

$$\varkappa_1 = \varkappa_0 \frac{\delta - \delta_{cr}}{2R - \delta_{cr}}.$$

To determine the turbulence constant $\kappa_0$ we use the suggestion [7] that the ratio of coefficients of the turbulent exchange for the heat and momentum near the wall is equal to 1.08 and, as one moves away from the wall, it increases to 1.5. In other words, the intensity of turbulent exchange far from the wall (on the outer boundary of the layer) is a factor of 1.5/1.08 = 1.39 higher than near the wall. As the thickness of the layer (as the scale of vortex motion) increases up to the thickness of the channel, the intensity of turbulent exchange (with relative value 1.39) is superimposed on the intensity of the turbulent exchange near the wall (the relative value 1.0), and causes an increase of the turbulent transport at the gas—liquid boundary by a factor 1.39 + 1.0 = 2.39, respectively, i.e., the coefficient $\kappa_1$ becomes equal to 2.39 $\kappa_0$.

A provisional value of $\kappa_1$ can be obtained theoretically by applying another Prandtl hypothesis (about the path length of mixing in the near-wall region), and by assuming that the tangential stress at the outer boundary of the viscous sublayer must be the same if the calculation is carried out by either hypothesis:

$$\rho\varkappa^2\delta_m^2(du/dy)^2 = \rho\varkappa_1\delta_m u_m du/dy.$$

Hence, assuming that $u_m = \delta_m du/dy$ we find $\kappa_1 = \kappa^2 = 0.4^2 = 0.16$. However, the experiment shows that in the absence of a rigid wall at the boundary of the gas flow (it is replaced by the pliable liquid film) in a developed turbulent flow of gas along the film of the liquid $\kappa_1 = 0.115$. Accordingly, $\kappa_0 = \kappa_1/2.39 = 0.048$. Thus, the coefficient $\kappa_1$ takes the following values depending on the quantity $\delta$:

$$\varkappa_1 = \begin{cases} 0 & \text{for} \quad 0 < \delta \leqslant \delta_{cr}, \\ \varkappa_0 \dfrac{\delta - \delta_{cr}}{2R - \delta_{cr}} & \text{for} \quad \delta_{cr} < \delta \leqslant 2R, \\ 2{,}39\varkappa_0 & \text{for} \quad \delta > 2R. \end{cases}$$

If we know $D_T$ and the heat capacity of the saturated gas at the temperature of the liquid $c_{gf} = (c_{pg} + c_v d_f)/(1 + d_f)$, we find the turbulent heat conductivity $\lambda_T = \rho c_{gf} D_T$. It should be noted that in the determination of the mass flux of vapor near the surface of the liquid, one introduces a correction for the Stefan current $\Psi = 1 + d_f$ for that (conventional) part of the flow which is caused by molecular diffusion. For the part of the vapor current caused by the turbulent diffusion, this correction is meaningless because there is no sharp distribution between the counterflow of the first and second components, since there exist a fluctuation and vortex motion of volume structures which include simultaneously the molecules of both vapor and gas.

After determining the heat flux $Q_i$ between the gases and liquids according to [2], we found the average enthalpy of the gas $I_2$ in the end of the segment and according to the parameters: $t_{2w}, d_{2w}, t_2, d_2, I_{ef}, t_{ef}, d_{ef}$, as well as the flow rate of the gas in the layer and core:

$$G_b = b\rho_g \int_0^{\delta_w} u dy = b\rho_g (7/8) u_a \zeta^{1/7}\delta_w; \quad G_a = G_g - G_b.$$

From the balance equation, analogously to the calculation of the laminar layer but using n = 1/7 for the profiles of the velocity and enthalpy, we find for the enthalpy in the core of the flow for a turbulent boundary layer:

$$I_{ea} = \frac{I_2 G_g - (1/9) G_b I_{ef}}{G_g - (1/9) G_b}.$$

The local gas parameters $t_{ew}$, $d_{ew}$, $t_{ea}$, $d_{ea}$ (in the saturation boundary and in the core) corresponding to this enthalpy can be determined using the method [2].

After the turbulent boundary layers join when $\delta = \delta_w = R$ and $G_b = G_g$, the enthalpy of the gas on the axis of the flow in the channel is determined from the formula

$$I_{ea} = \frac{I_2 G_g - 0{,}11 G_b I_{ef}}{G_g - 0{,}11 G_b} = 1{,}12 I_2 - 0{,}124 I_{cf}.$$

The gas velocity on the axis of the flow is $u_a = (8/7)\bar{u}$. The temperature $t_{ea}$ of the gas measured by a dry thermometer on the axis of the flow after the layers join can be determined in accordance with the adopted power-law distribution:

$$\frac{t - t_{ef}}{t_{ea} - t_{ef}} = \left(\frac{y}{R}\right)^{1/7}.$$

From the expression for the average temperature of the gas

$$t_2 = \frac{1}{R}\int_0^R t\,dy$$

we then obtain

$$t_{ea} = (8/7)\,t_2 - (1/7)\,t_{ef}.$$

The remaining parameters are determined by using the method [2]. The parameters of the media at the end of the segment $\Delta x_i$ are the initial values for the subsequent computational segment $\Delta x_{i+1}$, and the calculation is repeated.

Using the developed method, we developed an algorithm and calculated the heat and mass exchange in a planar channel for a direct comparison with the experimental data of [1] (experiment no. 6). It is seen from Fig. 4, the calculation gives a good agreement with the experimental data, and the calculated average temperatures of the liquid and gas (measured by a wet thermometer) coincide, for practical purposes, completely with the experimental values.

## NOTATION

$\rho$, $\rho_g$, $\rho_v$, densities of the humid gas, dry gas, and the vapor; $\bar{u}$, $u$, $u_a$, $u_m$, relative average velocity, relative instantaneous velocities and the relative velocities in the core of the flow and at the boundary of the viscous sublayer; $\Delta x_i$ and b, length and width of the computational segment; $Q_i$, $G_{vi}$, heat and mass fluxes on the computational segment of the channel; Q and $G_v$, total heat and mass fluxes; t, d, and I, temperature of the medium, the absolute humidity of the gas and its specific enthalpy; $t_{bf}$, initial temperature of the liquid with which it enters the volume of the reactive space on the segment under consideration; $\nu$, kinematic viscosity of the gas; $\lambda$, $\lambda_T$, D, $D_T$, molecular and turbulent heat conductivities and the diffusion coefficient; $I_v = r_0 + c_v t_f$, enthalpy of the vapor; $r_0$, heat of vapor formation at the melting-solidification temperature; $\delta$, $\delta_f$, $\delta_w$, $\delta_c$, thicknesses of the boundary hydrodynamic layer ($\delta_0$ refers to the previous segment), of the liquid film, and of the layers of saturated and unsaturated gas; $\delta_m$, $\delta_{cr}$, thicknesses of the viscous sublayer and of the hydrodynamic boundary layer when it becomes turbulent, respectively; $\delta_1 \approx \delta/3$, displacement thickness (for the laminar layer); $\zeta = \delta_w/\delta$, a ratio; $d_e$ and R, equivalent diameter and half the channel thickness for the passage of the gas; $G_g$ and $G_f$, flow rates of dry gas and liquid in the channel; $G_a$ and $G_b$, flow rates of dry gas in the core of the flow and in the layer of saturated gas; $c_{pg}$, $c_v$, $c_f$, specific heat capacities of the dry gas, vapor and liquid. The indices f, w and a refer to the initial parameters of the medium on the entire segment $\Delta x_i$ at the gas–liquid boundary, on the boundary between the saturated and unsaturated gas, and in the core of the flow, respectively, e refers to these parameters at the end of the segment, and 1 and 2 to the average parameters at the beginning and end of the segment.

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